

一. 通信系统的性能度量

1. 模拟系统

1) 有效性: 带宽

2) 可靠性: 信噪比 \rightarrow 信噪比增益.

2. 数字系统

1) 有效性: 频带利用率: R_b or R_s/B .

R_b (b/s = bps): 信息传输速率/信息速率/比特率; T_b (s): bit 间隔, $R_b = \frac{1}{T_b}$.

R_s (symbol/s = baud): 码元传输速率/符号速率; T_s (s): 符号间隔, $R_s = \frac{1}{T_s}$.

各电平: $R_b = R_s \cdot H$ 符号率 $R_s \cdot \log_2 M$.

多次电平变换/bit: $R_b = R_s/N$.

2) 可靠性: 信噪比 \rightarrow 误比特率 P_b or 误码率 (误符号率) P_s ; E_b or E_s/n_0 .

P_b : wrong bits / total bits, 二进制: $P_b = P_s$

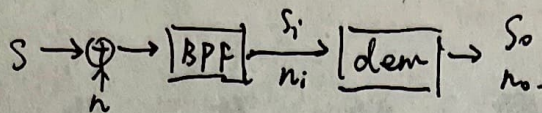
P_s : wrong symbols / total symbols, M进制: $P_b < P_s$.

自然码: $P_b = \frac{M}{2M-1} P_s$

格雷码: $P_b \approx \frac{1}{\log_2 M} P_s$

折叠码: ...

二. 模拟系统.



1. AM ($\beta_{AM} \leq 1$).

1) 有效性: $B = 2f_m$, $\eta_{AM} = \frac{f^2(t)}{A_0^2 + f^2(t)} \xrightarrow{\text{平均}} \frac{A_0^2/2}{A_0^2 + A_0^2/2} \leq \frac{1}{3}$

2) 可靠性: 包络检波 (小信号): 有门限效应

$$S_i = \frac{1}{2}A_0^2 + \frac{1}{2}E[f^2(t)]$$

$$N_i = n_o B = 2n_o f_m$$

$$S_o = E[f^2(t)]$$

$$N_o = n_o B = 2n_o f_m$$

$$G_1 = \frac{2E[f^2(t)]}{A_0^2 + E[f^2(t)]} \quad \text{单频, } \beta_{AM} = 1 \quad \frac{2}{3}$$

相干解调

$$S_i = \frac{1}{2}A_0^2 + \frac{1}{2}E[f^2(t)]$$

$$N_i = n_o B = 2n_o f_m$$

$$S_o = \frac{1}{4}E[f^2(t)]$$

$$N_o = \frac{1}{4}n_o B = \frac{1}{2}n_o f_m$$

$$G_2 = G_1.$$

2. DSB.

1) 有效性: $B = 2f_m$, $\eta = 100\%$.

2) 可靠性: 相干解调

$$S_i = \frac{1}{2}E[f^2(t)]$$

$$N_i = n_o B = 2n_o f_m$$

$$S_o = \frac{1}{4}E[f^2(t)]$$

$$N_o = \frac{1}{4}n_o B = \frac{1}{2}n_o f_m$$

$$G = 2$$

包络检波 (捕强载波).

3. SSB.

1) 有效性: $B = f_m$, $\eta = 100\%$.

2) 可靠性: 相干解调

$$S_i = \frac{1}{4}E[f^2(t)]$$

$$N_i = n_o B = n_o f_m$$

$$S_o = \frac{1}{16}E[f^2(t)]$$

$$N_o = \frac{1}{4}n_o B = \frac{1}{4}n_o f_m$$

$$G = 1$$

包络检波 (捕强载波).

4. VSB: $H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c) = \text{const}$

$$|\omega| \leq 2\pi f_m.$$

1) 有效性: $B \geq f_m$, $\eta = 100\%$.

2) 可靠性: 相干解调

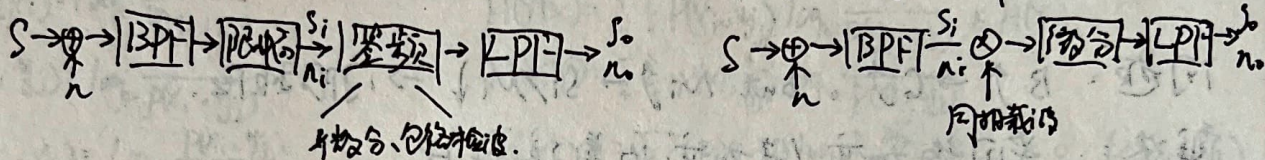
包络检波 (捕强载波).

5. FM: $S_{FM}(t) = A \cos[\omega_c t + K_{FM} \int f(t) dt]$ PM: $S_{PM}(t) = A \cos[\omega_c t + K_{PM} f(t)]$

宽带调制时 ($\beta \gg 1$): FM 带宽不随 ω_m 变, 优于 PM, 故一般用 FM.

1) 有效性: $B \approx 2(f_m + \Delta f_{max})$

2) 可靠性: 由于调制 (小噪声): 有门限效应. 宽带调制的相干解调.



$S_i = \frac{1}{2} A^2$

$N_i = n_0 B$

$S_o = \frac{K_{FM}}{4\pi^2} E[f^2(t)]$

$N_o = n_0 B \int_{f_m}^{f_m} |H(\omega)|^2 \frac{n_0}{(2\pi A)^2} dt = \frac{2n_0 f_m^3}{3A^2}$

$G = 3 \left(\frac{\Delta f_{max}}{f_m} \right)^2 \frac{E[f^2(t)]}{f(t)_{max}^2} \left(\frac{B}{f_m} \right)$

$S_i = \frac{1}{2} A^2$

$N_i = n_0 B = 2n_0 f_m$

$S_o = \frac{A^2 K_{PM}^2}{4} E[f^2(t)]$

$N_o = \int_{f_m}^{f_m} [n_0 \pi f^2] df = \frac{2n_0 \pi^2 f_m^3}{3}$

$G = 6 \left(\frac{\Delta f_{max}}{f_m} \right)^2 \frac{E[f^2(t)]}{f(t)_{max}^2}$

① 单频 $S(t) = A \cos[\omega_c t + K_{FM} \int A_m \cos \omega_m t dt] = A \cos[\omega_c t + \frac{K_{FM} A_m}{\omega_m} \sin \omega_m t] = A \sum_{n=-\infty}^{\infty} J_n(\beta_{FM}) \cos(\omega_c + n\omega_m)t$

$S(\omega) = \pi A \sum_{n=-\infty}^{\infty} J_n(\beta_{FM}) [\delta(\omega - \omega_c - n\omega_m) + \delta(\omega + \omega_c + n\omega_m)]$

有效性: $B = 2(f_m + \Delta f_{max}) = 2(1 + \beta_{FM}) f_m$, $\eta_{FM} = 1 - J_0^2(\beta_{FM})$ ($\sum_{n=0}^{\infty} J_n^2(\beta_{FM}) = 1$)

可靠性: $G_1 = 3\beta_{FM}^2(\beta_{FM} + 1)$

② 宽带 $S(t) = A \cos \omega_c t - [A K_{PM} \int f(t) dt] \sin \omega_c t$, $|K_{PM} \int f(t) dt|_{max} \leq \frac{\pi}{6}$

$S(\omega) = \pi A [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2} A K_{PM} \left[\frac{F(\omega - \omega_c)}{\omega - \omega_c} - \frac{F(\omega + \omega_c)}{\omega + \omega_c} \right]$

有效性: $B \approx 2f_m$

可靠性: $G_1 \approx 6 \left(\frac{\Delta f_{max}}{f_m} \right)^2 \frac{E[f^2(t)]}{f(t)_{max}^2}$, $G_2 \approx G_1$

③ 宽带单频: $S(t) = A \cos \omega_c t + \frac{1}{2} \frac{A A_m K_{FM}}{\omega_m} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$, $\beta_{FM} \leq \frac{\pi}{6}$

$S(\omega) = \pi A [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2} \frac{\pi A A_m K_{FM}}{\omega_m} [\delta(\omega - \omega_c + \omega_m) + \delta(\omega + \omega_c - \omega_m) - \delta(\omega - \omega_c - \omega_m) - \delta(\omega + \omega_c + \omega_m)]$

有效性: $B \approx 2f_m$, $\eta_{FM} = 1 - J_0^2(\beta_{FM})$

可靠性: $G_1 \approx 3\beta_{FM}^2$, $G_2 \approx G_1 = 3 A$

④ 宽带 $|k_{FM}(t)|_{max} \gg \omega_m$

有效性: $B \approx 2\Delta f_{max}$

可靠性: $G_1 \approx 6 \left(\frac{\Delta f_{max}}{f_m}\right)^3 \frac{E_b(f)}{H(f)_{max}}$

⑤ 宽带单频 $\beta_{FM} \gg 1$

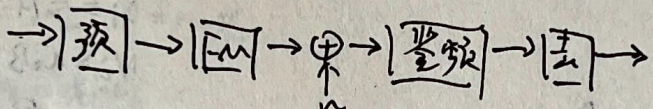
$$S(\omega) = \pi A \sum_{n=-\infty}^{\infty} J_n(\beta_{FM}) [\delta(\omega - \omega_c - n\omega_m) + \delta(\omega + \omega_c + n\omega_m)]$$

有效性: $B \approx 2\Delta f_{max}$, $\eta_{FM} = 1 - J_0^2(\beta_{FM})$

可靠性: $G_1 \approx 3\beta_{FM}^3$

问题: $B \uparrow \rightarrow G \uparrow$, but $N_i \uparrow \rightarrow S_i/N_i \downarrow \Rightarrow$ 门限效应.

解决: 采用预加重/去加重改善信噪比.



为提高输出信噪比, 针对鉴频输出端 $\varphi_n(f) \propto f^2$, 可采取

$$\begin{cases} \text{预: } H(\omega) = j\omega \\ \text{去: } H(\omega) = \frac{1}{1 + j\omega/\omega_1} \end{cases}$$

使高频信噪比改善.

三. 信息论.

1. 离散信源

信息量: bit

"事件" $I(x_i) = \log_2 \frac{1}{P(x_i)}$

"条件" $I(y_j | x_i) = \log_2 \frac{1}{P(y_j | x_i)}$

"联合" $I(x_i, y_j) = \log_2 \frac{1}{P(x_i, y_j)}$

"互" $I(x_i, y_j) = \log_2 \frac{P(x_i, y_j)}{P(x_i) P(y_j)}$

信息量的期望: 熵: bit/symbol.

$H(X) = \sum_i P(x_i) \log_2 \frac{1}{P(x_i)} \leq \log_2 M$

$H(Y|X) = \sum_i \sum_j P(x_i, y_j) \log_2 \frac{1}{P(y_j | x_i)} \leq H(Y)$

$H(X, Y) = \sum_i \sum_j P(x_i, y_j) \log_2 \frac{1}{P(x_i, y_j)}$

$K(X, Y) = \sum_i \sum_j P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i) P(y_j)} \geq 0 \cdot (\ln x < x-1)$

2. 离散信道 (对称)

1) $H(Y|X) = \sum_i \sum_j P(x_i, y_j) \log_2 \frac{1}{P(y_j | x_i)}$

$= \sum_i \sum_j P(x_i) P(y_j | x_i) \log_2 \frac{1}{P(y_j | x_i)}$

$= \sum_i P(x_i) \sum_j P(y_j | x_i) \log_2 \frac{1}{P(y_j | x_i)}$

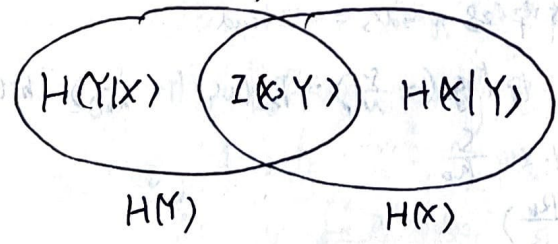
$\stackrel{\text{对称}}{=} \sum_i P(x_i) \cdot \text{const}$

$= \text{const} = \text{bit/symbol}$ (与 $P(y_j | x_i)$ 有关)

2) $C_{\text{max}} = \max \{ H(Y) - H(Y|X) \}$

$\stackrel{\text{对称}}{=} \log_2 M - \sum_j P(y_j) \log_2 \frac{1}{P(y_j)}$ bit/symbol...

3) $P_e \leq e^{-nE(R_0)}$. n : 总长; $E(R_0)$: 汉明距离; $R_0 \rightarrow C, n \rightarrow \infty, P_e \rightarrow 0$



e.g. 对称二进信道: $C_{\text{max}} = 1 - \epsilon \log_2 \epsilon - (1-\epsilon) \log_2 (1-\epsilon)$

3. 连续信源

绝对熵 $H(X) = \int_{-\infty}^{\infty} p(x) \log_2 \frac{1}{p(x)} dx + \log_2 dx$ 不随坐标变换改变。

相对熵 $H(X) = \int_{-\infty}^{\infty} p(x) \log_2 \frac{1}{p(x)} dx$; bit/symbol

$$\begin{cases} \text{功率受限} \\ \leq \log_2 \frac{1}{\sigma^2} e & , p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \\ \text{峰值受限} \\ \leq \log_2 2A & , p(x) = \frac{1}{2A} \end{cases}$$

"条件" $H(X|Y) = \iint p(x,y) \log_2 \frac{1}{p(x|y)} dx dy \leq H(X)$

"联合" $H(X,Y) = \iint p(x,y) \log_2 \frac{1}{p(x,y)} dx dy$

"互" $I(X,Y) = \iint p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)} dx dy \geq 0$

4. 连续信道 (与信源独立的 AWGN)

$$\begin{aligned} 1) H(Y|X) &= \iint p(x,y) \log_2 \frac{1}{p(y|x)} dx dy \\ &= \iint p(x) p(y|x) \log_2 \frac{1}{p(y|x)} dx dy \\ &= \int p(x) dx \int p(y|x) \log_2 \frac{1}{p(y|x)} dy \\ &= \int p(x) dx \int f(n) \log_2 \frac{1}{f(n)} dn \\ &= H(N) \text{ bit/symbol} \text{ 仅与 } n \text{ 有关} \end{aligned}$$

$$\begin{aligned} 2) C_{max} &= \max \{ H(X) - H(Y|X) \} \\ &= \max \{ H(X) - H(N) \} \\ &= \log_2 \sqrt{2\pi e(S+N)} - \log_2 \sqrt{2\pi eN} \\ &= \frac{1}{2} \log_2 \left(1 + \frac{S}{N} \right) \text{ bit/symbol} \end{aligned}$$

3) Nyquist 采样时, 采样率 $2B$ symbol/s = $2B$ band

$$R_{b,max} = C_{max} \cdot 2B = B \log_2 \left(1 + \frac{S}{N} \right) = B \log_2 \left(1 + \frac{S}{N_0 B} \right) \text{ bit/s}$$

$$\textcircled{1} R_b \xrightarrow{B \rightarrow \infty} \frac{S}{N_0} \log_2 e = 1.44 \frac{S}{N_0}$$

$$\textcircled{2} \frac{R_b}{B} = \log_2 \left(1 + \frac{E_b}{N_0} \cdot \frac{R_b}{B} \right)$$

$$\frac{E_b}{N_0} = \frac{2^{\frac{R_b}{B}} - 1}{\frac{R_b}{B}} = \frac{e^{\frac{R_b}{B} \ln 2} - 1}{\frac{R_b}{B}} \xrightarrow{R_b/B \rightarrow 0} \ln 2 = 1.643$$

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四. 信源编码 → 提高有效性

1. 内容.

① 抽样:

自然: 乘周期脉冲: 频谱单元冲不变.

$$x_s(t) = x(t) \cdot \sum_n \delta(t - nT_s)$$

采样: 抽样保持: 频谱单元冲变: 引起失真 → 补偿滤波器.

$$x_s(t) = [x(t) \cdot \sum_n \delta(t - nT_s)] * \text{Rect}(T_0)$$

低通:

$$f_s \geq 2f_H$$

带通: $f_L + kf_s \leq f_L$; $-f_H + (k+1)f_s \geq f_H \Rightarrow$

$$f_s \geq \frac{1}{\lfloor \frac{f_H}{f_H - f_L} \rfloor} \cdot 2f_H$$

② 量化:

1) 给定 $p_x(x)$, L , 求使 $\sigma_q^2 = \int_{x_{k-1}}^{x_k} (x - y_k)^2 p_x(x) dx$ 最小的“分层” $\{x_k\}$, “量尺” $\{y_k\}$.

$$\frac{\partial \sigma_q^2}{\partial x_k} = 0 \quad : \quad x_k = \frac{1}{2}(y_k + y_{k+1})$$

$$\frac{\partial \sigma_q^2}{\partial y_k} = 0 \quad : \quad y_k = \int_{x_{k-1}}^{x_k} x p_x(x) dx / \int_{x_{k-1}}^{x_k} p_x(x) dx \quad L \gg 1 \approx \frac{1}{2}(x_k + x_{k+1})$$

$\sigma_q^2 \ll 1$ 不过载: $\int_{-V}^V (x_{k+1} - x_k)^2 p_x(x) dx$

过载: $\int_{-V}^V (x+V)^2 p_x(x) dx + \int_V^{+\infty} (x-V)^2 p_x(x) dx$

2) 给定 $p_x(x)$, L , 求使 $\sigma_q^2 \ll 1$ $\int_{-V}^V (x_{k+1} - x_k)^2 p_x(x) dx \approx \frac{\Delta^2}{12} \int_{-V}^V [f'(x)]^2 p_x(x) dx$ 最小 $f(x)$

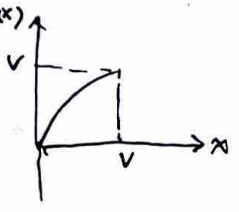
$$z = f(x), \quad \Delta z = \Delta = \frac{2V}{L}, \quad \frac{\Delta z}{\Delta x} \ll 1 \approx f'(x)$$

$$\min \left\{ \frac{\Delta^2}{12} \int_{-V}^V [f'(x)]^2 p_x(x) dx \right\} \stackrel{f(x)}{=} \min \frac{\Delta^2}{12} \int_{-V}^V [f'(x)]^2 p_x(x) dx$$

$$\frac{1}{2} \int_{-V}^V f'^2 dx \stackrel{f(x)}{=} \int_0^V f'(x) dx = V$$

$$f(x) = V \cdot \frac{\int_{-x}^x p_x^{\frac{1}{2}}(\alpha) d\alpha}{\int_0^x p_x^{\frac{1}{2}}(\alpha) d\alpha}$$

$$\sigma_q^2 = \frac{2}{3L^2} \left[\int_0^V p_x^{\frac{1}{2}}(\alpha) d\alpha \right]^2$$



a. 均匀分布 $p_x(x) = \frac{1}{2L} \Rightarrow f(x) = x$

$$\sigma_q^2 = \frac{V^2}{3L^2} = \frac{\Delta^2}{12} \cdot \text{SNR} \cdot \frac{S = V^2/3}{L^2}$$

b. Laplace 分布 $p_x(x) = \frac{1}{\sqrt{2}\sigma_x} \exp(-\frac{\sqrt{2}}{\sigma_x}x) \Rightarrow f(x) = V \frac{1 - \exp(-\sqrt{2}x/3\sigma_x)}{1 - \exp(-\sqrt{2}V/3\sigma_x)}$

$$\sigma_q^2 = \frac{9\sigma_x^2}{2L^2} [1 - \exp(-\sqrt{2}V/3\sigma_x)]^3, \text{SNR} \frac{S = \sigma_x^2}{9} L^2 [1 - \exp(-\sqrt{2}V/3\sigma_x)]^3$$

c. 均匀分布的均匀量化: $f(x) = x$

$$\sigma_q^2 = \frac{V^2}{3L^2} = \frac{\Delta^2}{12}, \text{SNR} \frac{S = \sigma_x^2 \triangleq \sqrt{D}V}{3D^2L^2} \approx 4.77 + 10 \lg D + 6.02n - (D: \text{动态范围})$$

d. 均匀分布的对数量化: $f(x) = \frac{1}{B} \ln x$

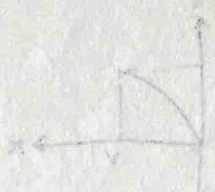
$$\sigma_q^2 = \frac{1}{6} \Delta^2 B^2 \int_0^V x^2 p_x(x) dx = \frac{1}{6} \Delta^2 B^2 \sigma_x^2, \text{SNR} \frac{S = \sigma_x^2}{\sigma^2 B^2} = \frac{3L^2}{13^2 V^2}$$

③ 编码:

1 symbol (L 电平) $\xrightarrow[\text{码字}]{\text{二进制}} \log_2 L \text{ bit.}$

2. 有效性: $R_b = f_s \cdot \log_2 L$

3. 可靠性: $\text{SNR} = \sigma_x^2 / \sigma_q^2$



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五. 信道编码 → 提高可靠性

1. 基带编码: 代路码型

① 内容

1) 无冗余二元

不归零: 功率谱集中 > 归零: 可提取时钟

单极性: 易实现 ; 双极性: 低直流含量

自然: -- ; 差分: 克服极性模糊, 引入有限长码字扩散.

2) 二元效率二元

Manchester: 1 → 01 ; 0 → 10

CDP: 逢 1 改变 → 01 或 10

CMI: 1 → 交替 00, 11 ; 0 → 01

Miller: 在 Manchester 的上升沿变

3) 三元, 归零

AMZ: 1 → 交替 1, -1

HDB3: 0000 → 000V 或 000V, V 及 环极性交替, 两 V 间奇数 B.

② 有效性

1) 码率: $\eta = \frac{R_b}{R_c} = \frac{1}{N} \frac{\log M}{\log 2}$ (对 3); $\eta = \frac{R_b}{R_c} = \frac{1}{N}$ (对 2).

2) 功率谱: $S(j\omega) = \sum_k R(k) e^{j\omega k} = \sum_k E \{ a_n a_{n+k} \} e^{-j\omega k}$.

③ 可靠性

1) 定时、抖动

2) 纠错检错

3) 误码扩散.

2. 差错控制编码

① 内容

线性 / 非线性: 信息码与监督码的检验关系.

分组 / 卷积: 信息码与监督码的约束方式. (D 越大越后(右), 卷积(左), 分组(右), 合理)

完备 / 不完备: $2^{n-k} \geq \sum_{i=0}^k C_n^i$

系统 / 非系统: 信息位编码后是否改变.

伴随纠错 / 纠突发错.

1) Hamming ($2^r - 1, 2^r - 1 - r, d=3$): 线性分组码.

编: $A = M \cdot G, G = [I_k, Q]$

译: $S = (A+E)H^T = EH^T, H = [P, I_r],$ 根据 S 纠错.

2) 奇偶监督 ($n, n-1, e=奇数$): 线性分组.

编: $A = (m, \sum_{i=1}^n m_i)$

译: $S = \sum_{i=1}^n a_i$

3) 群计数

编: $A = (m, \sum_{i=1}^n (m_i = 1))$

译: 检除 0 或对称变换外的所有错.

4) 循环

($n, k, weight \{g(D)\}$): 线性分组, 随机/突发.

编: $A(D) = D^{n-k} M(D) + N(D), N(D) = D^{n-k} M(D) \text{ mod } g(D)$

$A(D) = M(D) \cdot G(D), G(D) = [D^{n-k+i} + D^{n-k+i} \text{ mod } g(D)]_{i=0 \sim k-1}$

译: $S(D) = [A(D) + E(D)] \text{ mod } g(D) = E(D) \text{ mod } g(D),$ 由接收码 $+ E(D)$ 试出发生一位错时的位置

$S(D) = [A(D) + E(D)] \cdot h^T(D) = E(D) \cdot h^T(D), h(D) = (D^n + 1) / g(D)$

5) BCH

($2^m - 1, \geq 2^m - 1 - mt, \geq 2t + 1$) 中等号时为循环汉明码.

编: 由 m, t 查表得 $S(D) = \text{LCM}[m_1(D), m_2(D), \dots, m_{2t+1}(D)]$. 每个因式 m_i 阶.

译: 彼得森.

6) 交织

M 到 n 行, $t = t_0 \cdot M, M$ 为交织深度, 纠突发错.

编: 列入行出 - 延时 $M \cdot N$

译: 行入列出, 延时 $M \cdot N$.

7) 卷积

(n, k, N) 状态数 $2^{k(N-1)}$; 约束长度/寄存器个数: N ; 线性, 卷积.

编: $x_i(D) = M(D) \cdot g_i(D),$ 将各 $x_i(D) (i=1 \sim n)$ 位交错排列得 $X(D)$

$X(D) = M(D) \cdot G(D),$ 将各 $g_i(D)$ 每行延 k 位交错排列得 $G(D)$.

译: 流源等概, 二进制对称信道, 最大似然 = 最小汉明距: viterbi

② 有效性: 码率 $\eta = \frac{R_0}{R_b} = \frac{k}{n}$.

问题: 增加频带利用率

解决: M 进制: $R_0'' = R_b' \cdot \log_2 M$.

③ 可靠性: 1) 最小码距: $d_{min} \geq 2t + 1 / e + 1 / t + e + 1 (e > t)$

e.g. 线性码: 最小码距 = 最小码重

2) 编码增益: P_s - 定时采用纠错编码后所需的 $\frac{E_b}{N_0}$.

e.g. 分组码: $P_s' = \frac{1}{n} \sum_{i=1}^n C_n^i P_s(\frac{E_b}{N_0} \cdot \frac{k}{n}) (1 - P_s(\frac{E_b}{N_0} \cdot \frac{k}{n}))^{n-i}$

问题: 连 0 连 1 引起的定时误差.

解法: 1) 加扰 LIR: $G = S / \sum C_i D_i +$ 解扰 FIR: $S = \sum C_i D_i \cdot G$: 有限差错扩散

2) M 序列发生器: $g(D) = \sum C_i D_i, 2^n - 1$ 周期; $R_g(m) = \begin{cases} 2^n - 1 & m=0 \\ -1 & m \neq 0 \end{cases}$ ($weight \{g(D)\}$).

2^{n-1} 个 1; $2^{n-1} - 1$ 个 0, 2^{n-1} 个游程.

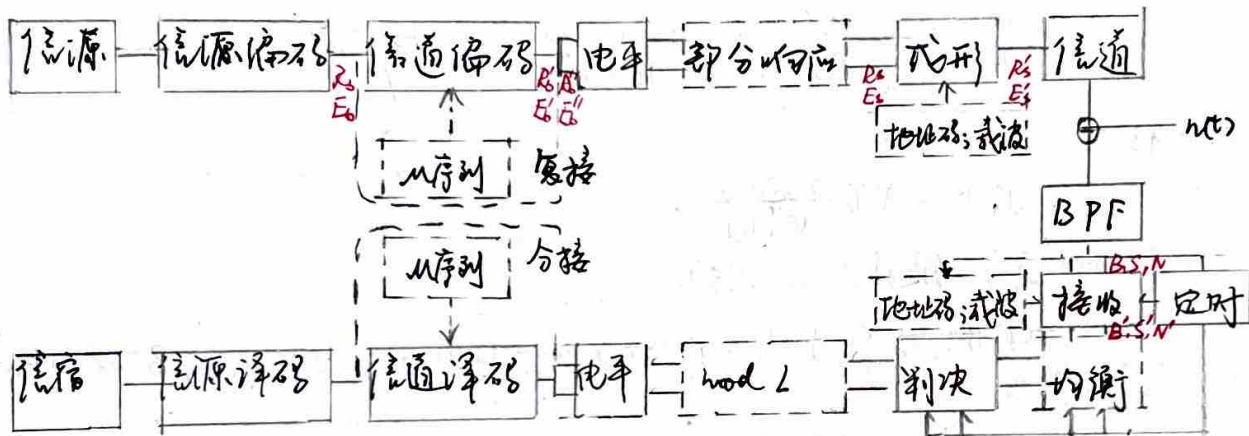
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六. 数字系统: 发端



1. 部分响应系统

① 内容: $\frac{d_n}{L}$ 预编码, mod L $\xrightarrow{\text{FIR}}$ $\frac{a_n}{L}$ 相关编码 $\xrightarrow{\text{FIR}}$ $\frac{c_n}{2L-1}$ $\xrightarrow{\text{mod L}}$ $\frac{d_n}{L}$

$$a_n = (d_n - \sum_{k=1}^M h_k a_{n-k}) \text{ mod } L \quad \text{eg 第2类: } 1+D; \text{ 第2V类: } 1-D^2$$

$$c_n = a_n + \sum_{k=1}^M h_k a_{n-k}$$

$$d_n = c_n \text{ mod } L$$

② 有效性: 加快成码后码元频谱的衰减。

③ 可靠性: 1) 由于频域滤波, 故时域展宽, 由定时误差导致的码间串扰减少。
2) 进制数越多, 误码率升高, 见后。

2. 成形: 基带、带通

① 内容:

带通占用信号:

能量:

瞬时功率:

平均功率:

相关:

距离:

$$s(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT_s)$$

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} s_n \cdot f_n(t), \quad s_n = \int_{-\infty}^{\infty} \hat{s}(t) f_n(t) dt, \quad \text{单位完备正交基 } \{f_n(t)\}_n$$

$$E_s = \sum_{n=-\infty}^{\infty} s_n^2 = \int_{-\infty}^{\infty} \hat{s}^2(t) dt$$

$$S(t) = \hat{s}^2(t)$$

$$S = E_s / T_s = E_s \cdot R_s$$

$$P_{mk} = \int_{-\infty}^{\infty} \hat{s}_m(t) \hat{s}_k(t) dt / \sqrt{E_m \cdot E_k}$$

$$d_{mk} = \left[\int_{-\infty}^{\infty} [\hat{s}_m(t) - \hat{s}_k(t)]^2 dt \right]^{1/2} = (E_m + E_k - 2\sqrt{E_m E_k} P_{mk})^{1/2} \quad \underline{E_m = E_k = E} \quad \sqrt{2E(1 - P_{mk})}$$

② 有效性:

$$G_s(j\omega) = G_s(j\omega), G_c(j\omega), G_c(j\omega) \text{ 为 } g_c(t) \text{ 的频谱.}$$

基带:

$$R_s/B \leq 2 \text{ band/Hz} \quad R_s = R_{s,max}/n.$$

带通:

$$R_s/B \leq 1 \text{ band/Hz} \quad R_s' = (R_s/B) \cdot B' \quad (f_{at}) \approx \frac{1}{\text{向}} F(\frac{B'}{B}).$$

④

N路, M进制:

$$\frac{R_s}{R_b} \xrightarrow{\text{分}} \frac{R_s/N}{R_b/N} \xrightarrow{\text{调}} \frac{R_s/N \cdot \log_2 M}{R_b/N} \xrightarrow{\text{合}} \frac{R_s/\log_2 M}{R_b}.$$

1) 基带双极性:

$$G_s(f) = A^2 T_s \frac{\text{sinc}^2(kfT_s)}{(\pi f T_s)^2}.$$

2) MFSK (正交: $\Delta f = \frac{1}{2T_s}$): 键控/模拟调频

$$G_s(f) \propto G_s(f+f_1) + G_s(f-f_1) + \dots + G_s(f+f_m) + G_s(f-f_m), B = 2B_0 + f_m - f_1.$$

3) MASK: 键控/键控

$$G_s(f) \propto G_s(f+f_c) + G_s(f-f_c).$$

4) OOK: 键控

$$G_s(f) \propto G_s(f+f_c) + G_s(f-f_c).$$

5) MQAM, MOPR: 正交调制

$$G_s(f) \propto G_{2,Q}(f+f_c) + G_{2,Q}(f-f_c).$$

6) MPSK: 正交调制/键控/脉冲插入法/MDPSK

$$G_s(f) \propto G_s(f+f_c) + G_s(f-f_c) \quad \text{QPSK } G_s(f) \propto \frac{1}{f_0}, \frac{B_{R,max}}{B_{R,min}} = \infty, \Delta\varphi = 180^\circ.$$

7) OQPSK: 正交调制, Q路延时 $T_s/2$, 起升/起降

$$G_s(f) \text{ 同 OQPSK.}$$

$$\text{OQPSK } G_s(f) \propto \frac{1}{f_0}, \frac{B_{R,max}}{B_{R,min}} = \sqrt{2}, \Delta\varphi = 90^\circ.$$

8) MSK (正交: $\Delta f = \frac{1}{2T_s}$): 正交调制, Q路延时 T_s , 正弦成形/模拟调频, 频偏 $\pm \frac{1}{4T_s}$. ECPSK.

$$G_s(f) \text{ 谱零带宽为 QPSK 的 } 0.5 \text{ 倍, 效率更集中. } \propto \frac{1}{f_0}, \frac{B_{R,max}}{B_{R,min}} = 1, \Delta\varphi = 0^\circ.$$

③ 可靠性: 误码率

$$P_s = \sum P(s_i) \rightarrow P(e|s_i) = \sum P(s_i) \sum_{j \neq i} P(s_j).$$

→ 基带双极性:

一般FSK: 恒包络、宽带限带、起伏、窄带 $\xrightarrow{\text{类功放}}$ 恒包络、宽带。
 CPFSK: 恒包络、窄带功放, 恒包络、窄带。(相移连续可于时数越高带外衰减越大)

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③ 可靠性 (比特)

$$P_s = \sum_i P(s_i) p(e|s_i) \leq \sum_i P(s_i) \sum_{j \neq i} P(i \rightarrow j) \rightarrow \text{平均差错概率}$$

1) 基带双极性: 最佳接收:

$$P_{s1} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M-1}} \cdot \frac{E_s}{N_0}\right)$$

基带单极性: 最佳接收:

$$P_{s2} = \frac{2(M-1)}{M^2} Q\left(\frac{\pi}{4} \sqrt{\frac{6}{M-1}} \cdot \frac{E_s}{N_0}\right) \quad P_b \approx \frac{1}{\log_2 M} P_s$$

2) MFSK

最佳接收

$$P_{s1} \approx (M-1) Q\left(\sqrt{\frac{E_s}{N_0}}\right) \quad \text{正交}$$

$$m=2, \text{非正交: } P_s = Q\left(\sqrt{1.22} \frac{E_s}{N_0}\right)$$

非相干:

$$P_{s2} \approx \frac{M-1}{2} e^{-\frac{E_s}{N_0}} > P_{s1} \quad \text{正交}$$

$$P_b \approx \frac{M}{2(M-1)} P_s$$

3) MASK

最佳接收:

$$P_s = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M-1}} \cdot \frac{E_s}{N_0}\right)$$

$$P_b \approx \frac{1}{\log_2 M} P_s$$

4) OOK

最佳接收:

$$P_{s1} = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

包络检波:

$$P_{s2} \approx \frac{1}{2} e^{-\frac{E_s}{2N_0}} > P_{s1}$$

5) M-QAM

最佳接收:

$$P_{s1} = \frac{2(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{6}{M-1}} \cdot \frac{E_s}{N_0}\right)$$

M-QPSK

最佳接收:

$$P_{s2} = \frac{2(M-1)}{M} Q\left(\frac{\pi}{4} \sqrt{\frac{6}{M-1}} \cdot \frac{E_s}{N_0}\right) \quad P_b \approx \frac{1}{\log_2 \sqrt{M}} P_s$$

6) MPSK

最佳接收:

$$P_{s1} \approx 2Q\left(\sqrt{\frac{2E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}\right) \quad m=4 \text{ 时 } P_{s1} = 2Q\left(\frac{E_s}{N_0}\right) \left[1 - \frac{1}{4} Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]$$

M-DPSK

最佳接收:

$$P_{s2} \approx 2Q\left(\sqrt{\frac{2E_s}{N_0} \sin^2\left(\frac{\pi}{2M}\right)}\right)$$

M-DPSK

差分相干:

$$P_{s3} > P_{s2} > P_{s1} \quad m=2 \text{ 时 } P_{s3} \approx \frac{1}{2} e^{-\frac{E_s}{N_0}} \quad P_b \approx \frac{1}{\log_2 M} P_s$$

7) QPSK

最佳接收:

$$P_s = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) \left[1 - \frac{1}{4} Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]$$

8) MSK

最佳接收:

$$P_{s1} = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) \left[1 - \frac{1}{4} Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]$$

正交2PSK相干解调: P_{s2} 的 $\frac{E_s}{N_0}$ 比 P_{s1} 大 3dB.

正交2PSK非相干: P_{s3} 的 $\frac{E_s}{N_0}$ 比 P_{s2} 大 3.6dB.

差分解调: P_{s3} 在 DPSK 相干的相干和非相干的中间.

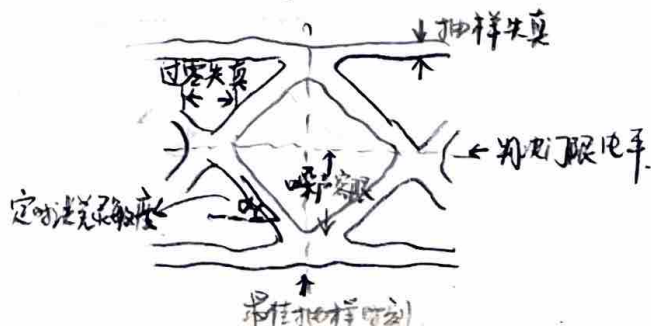
~~基带调制~~ 基带

七. 数字系统: 收端、最佳接收.

1. 均化器: 以时域均衡算法为例.

1) ISI \Rightarrow 用 $G_{22}(f)$ 作为 2R 均化器不稳定, 改用 1/2R. 峰值畸变: $D = \frac{1}{T} \sum_{k \neq 0} |H_k|$.

$$\begin{bmatrix} x_{-N} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ x_N & \dots & x_N & \\ 0 & & & \vdots \\ \vdots & & & 0 \\ 0 & \dots & 0 & x_N \end{bmatrix} \begin{bmatrix} c_N \\ \vdots \\ c_1 \end{bmatrix}_{(2N+1) \times 1} = \begin{bmatrix} y_{-2N} \\ \vdots \\ y_0 \\ \vdots \\ y_{2N} \end{bmatrix}$$



2) AWGN \Rightarrow ...

2. 最佳接收

1) AWGN \Rightarrow 判决规则: $I = \arg \max_i P(s_i(t) | x(t)) = \arg \max_i \frac{P(s_i(t) | x(t)) P(x(t) | s_i(t))}{P(x(t))}$, $i=1, \dots, M$, $t \in [0, T_s]$

设B为BPF带宽, 则 $\sigma_n^2 = N_0 B$, 采样点数 $k \geq 2BT_s$.

$$f(\vec{u}) = f(u_1) \dots f(u_k) = \frac{1}{(\sqrt{2\pi}\sigma_n)^k} \exp\left[-\frac{1}{2\sigma_n^2} \sum_{k=1}^k u_k^2\right]$$

$\therefore f_{\text{AWGN}}(x) \stackrel{k \rightarrow \infty}{\approx} \frac{1}{(\sqrt{2\pi}\sigma_n)^k} \exp\left\{-\frac{1}{N_0} \int_0^{T_s} [x - s_i(t)]^2 dt\right\} \sim$ 无穷维高维分布

误码率 P_e 可由 $f_{\text{AWGN}}(x)$ 积分得到, 对不同的判决方式有不同形式:

a. 若各符号等概: $P(s_i(t)) = P(s_j(t))$ $i \neq j$ $i, j=1, \dots, M$.

判决规则: $I = \arg \max_i P(x(t) | s_i(t)) = \arg \min_i \int_0^{T_s} (x - s_i)^2 dt = \arg \min_i \langle x - s_i | x - s_i \rangle$

等价形式: $I = \arg \min_i \sum_{k=1}^k \langle x - s_i | f_k \rangle \langle f_k | x - s_i \rangle = \arg \min_i \sum_{k=1}^k \langle x_k - s_{ik} | x_k - s_{ik} \rangle$

b. 若各符号等概且等能量: $\int_0^{T_s} s_i(t) dt = \int_0^{T_s} s_j(t) dt$ $i \neq j$ $i, j=1, \dots, M$.

判决规则: $I = \arg \max_i \int_0^{T_s} x(t) s_i(t) dt = \arg \max_i \langle x | s_i \rangle$

等价形式: $I = \arg \max_i \sum_{k=1}^k \langle x | f_k \rangle \langle f_k | s_i \rangle = \arg \max_i \sum_{k=1}^k \langle x_k | s_{ik} \rangle$

a, b情况下, $\{f_k\}_1^N$ 为单位完备正交基. b情况下, 接收机的等价形式是M路或N路匹配滤波器.

M路: $h_i(t) = s_i^*(T_s - t)$, $H_i(f) = S_i^*(f) e^{-j\omega T_s}$, $y_i(t) = \int_0^t x(\tau) s_i^*(T_s - t + \tau) d\tau \stackrel{t \leq T_s}{=} \int_0^{T_s} x(\tau) s_i^*(\tau) d\tau$

N路: $h_k(t) = s_{ik}^*(T_s - t)$, $H_k(f) = S_{ik}^*(f) e^{-j\omega T_s}$, $y_{ik}(t) = \int_0^t x(\tau) s_{ik}^*(T_s - t + \tau) d\tau \stackrel{t \leq T_s}{=} \int_0^{T_s} x(\tau) s_{ik}^*(\tau) d\tau$

采样点SNR:

$$\frac{S}{N} = \frac{\left| \int_{-\infty}^{\infty} x(f) H_i(f) e^{j\omega T_s} df \right|^2}{\int_{-\infty}^{\infty} |G_n(f)|^2 df} = \frac{\left| \int_{-\infty}^{\infty} |s_i(f)|^2 df \right|^2}{G_n(f)} = \frac{E_s}{N_0 T_s}$$

\therefore 接收机, BPF后 $\text{SNR} = \frac{E_s P_s}{N_0 B}$, $\frac{P_s}{B} \leq 2$.

\therefore 匹配滤波器达到了最大采样点SNR.

2) ISI \Rightarrow 用匹配滤波器时, $|H_i(f)| = |G_{\text{ISI}}(f)|^2$.

b:

